Continuum Sensitivity Analysis and Shape Optimization of Dirichlet Conductor Boundary in Electrostatic System

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This paper proposes an optimal design method of conductor shape in an electrostatic system by using the continuum shape sensitivity analysis. The continuum sensitivity formula for conductor shape with the Dirichlet boundary condition is analytically derived using the material derivative concept of continuum mechanics and the adjoint variable technique. The geometry change of the conductor surface is determined by the velocity field from the continuum sensitivity formula and it is expressed by the level set method. A simple numerical example is tested to show usefulness of the proposed method.

***Index Terms*—Continuum sensitivity, electrostatic system, finite element method, level set method, shape optimization.**

I. Introduction

In most of electrostatic systems, voltage sources are applied to conductor electrodes and the distribution of voltage and electric field inside the system is determined by the conductor geometry. So, the desired field distribution or the required performance in the electrostatic system can be obtained by optimizing the conductor geometry. For example, the field intensity near the electrode is alleviated to lower possibility of the electrical breakdown in high-voltage systems [1]. In a case of dielectrophoresis application, a non-uniform electric field is generated to guide dielectric particles in an intended direction [2].

In the boundary value problems of electrostatic system, the Dirichlet boundary condition is imposed on the conductor surface to analyze the field distribution. The geometry of the boundary conductor can be an arbitrary shape to give a desired field distribution. That is, because the design problem of the conductor boundary is a shape design problem, its design variable is not the size but the shape. Therefore, an optimization method based on the shape sensitivity analysis is suitable for this design problem. In particular, the continuum sensitivity analysis has some advantages for efficient optimization process.

Most of the continuum sensitivity analyses for electrostatic systems have been applied to the shape design of the material interface between different dielectrics [3], [4]. In this paper, the continuum sensitivity analysis is also employed to derive 3 dimensional shape sensitivity formula for the Dirichlet boundary deformation. The derivation procedure is based on the material derivative concept of continuum mechanics and the adjoint variable technique. The objective function is arbitrary function of voltage and field, which is defined in a region inside the system. Since the derived sensitivity formula is in a closed form, its numerical implementation is relatively simple and its numerical values is accurate. In addition, numerical calculation the sensitivity formula of the state and adjoint variables does not depend on the numerical analysis method.

The level set method is used to easily deal with the shape evolutions during the design process. The state and the adjoint variables are numerically calculated using the finite element method and are used for calculation of the velocity field for the optimization process. In this digest, a simple numerical example is tested to show usefulness of the proposed method.

II. Derivation of Continuum Sensitivity Formula

An electrostatic system with the conductor boundary is shown in Fig. 1, where the boundary of domain $\Omega = \Omega_r \cup \Omega_p$ is Γ_0 and Γ_1 with the Dirichlet and homogeneous Neumann conditions, respectively. There, ε is the permittivity and $\mathbf{n}$ is the unit normal vector. The objective function is defined as a regional integral of the electric potential V and the field $\mathbf{E}$ as follows:

$$\mathrm{F} = \int_\Omega g\left(V, \mathbf{E}(V)\right) \mathrm{m}_p d\Omega \tag{1}$$

where g is any differentiable function and m_p is the characteristic function for integral region of Ω_p. The state equation is a kind of the equality constraint in this optimization.

$$\mathrm{a}(V, \bar{V}) = \mathrm{l}(\bar{V}) \qquad \forall\ \bar{V} \in \Phi \tag{2}$$

where
$$\mathrm{a}(V, \bar{V}) = \int_\Omega \varepsilon \mathbf{E}(V) \cdot \mathbf{E}(\bar{V}) d\Omega \tag{3}$$

$$\mathrm{l}(\bar{V}) = \int_\Omega \rho \bar{V} d\Omega \tag{4}$$

In (2), (3) and (4), ρ is the space charge density, Φ is the space of admissible state variable, and the upper bar denotes the test function of corresponding equation. The state equation (2) is incorporated in the objective function (1) to give an augmented objective function.

$$\mathrm{G} = \mathrm{F} + \mathrm{l}(\bar{V}) - \mathrm{a}(V, \bar{V}) \qquad \forall\ \bar{V} \in \Phi. \tag{5}$$

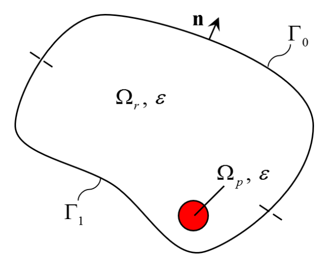

Fig. 1. Electrostatic system of Dirichlet conductor boundary problem.

The material derivative of the augmented objective function (5) is used to derive the shape sensitivity.

$$\dot{\mathrm{G}} = \dot{\mathrm{F}} + \dot{\mathrm{l}}(\bar{V}) - \dot{\mathrm{a}}(V,\bar{V}) \qquad \forall\ \bar{V} \in \Phi. \tag{6}$$

To express (6) in terms of velocity field, an adjoint variable equation is introduced.

$$\mathrm{a}(\lambda,\bar{\lambda}) = \int_{\Omega}\left(g_V\bar{\lambda} + \mathbf{g}_E \cdot \mathbf{E}(\bar{\lambda})\right)d\Omega \qquad \forall\ \bar{\lambda} \in \Phi \tag{7}$$

where λ is the adjoint variable, and the subscripts V and E indicate the partial derivatives with respect to V and $\mathbf{E}$. After some mathematical manipulations with (2) and (7), the shape sensitivity formula for the Dirichlet boundary deformation is derived as follows:

$$\dot{\mathrm{G}} = \int_{\Gamma_0} \varepsilon E_n(V)E_n(\lambda)\mathrm{V}_n\, d\Gamma \tag{8}$$

where $\mathbf{V}$ is the velocity on the conductor boundary and the subscript n denotes the normal component of the variable. If the velocity for the boundary deformation is taken as (9), since the value of the sensitivity is always negative, it can be applied to the minimization problem.

$$\mathrm{V}_n = -\varepsilon E_n(V)E_n(\lambda) \tag{9}$$

The shape evolution during the optimization process is expressed by using the level set equation [5].

$$\frac{\partial\phi}{\partial t} + \mathrm{V}_n\left|\nabla\phi\right| = 0 \tag{10}$$

where ϕ is the level set function and t is the time. The level set method and the velocity field (9) from the continuum sensitivity analysis are coupled by substituting (9) into (10). The coupling transforms the optimization process into a transient analysis.

III. Numerical Test

Fig. 2 shows a simple test model with the inner circular anode and the outer filleted rectangular cathode. The design objective is to find the optimal shape of the cathode for a uniform field distribution on Ω_p. In fact, the exact optimal shape of this simple design problem is known to be a circular concentric to the anode. The objective function to be minimized is taken as

$$\mathrm{F} = \int_{\Omega}\left[\mathbf{E}(V) - \mathbf{E}_0\right]^2 \mathrm{m}_p\, d\Omega \tag{11}$$

where $\mathbf{E}_0$ is the desired field whose intensity is constant in Ω_p. The adjoint equation for the objective function (11) is as follows:

$$\mathrm{a}(\lambda,\bar{\lambda}) = \int_{\Omega} 2\left[\mathbf{E}(V) - \mathbf{E}_0\right]\cdot\mathbf{E}(\bar{\lambda})\mathrm{m}_p\, d\Omega \qquad \forall\ \bar{\lambda} \in \Phi \tag{12}$$

In the optimization process of Fig. 3, the cathode surface is gradually evolved with time and finally it arrives to the expected circular shape. The objective function in Fig. 4 converges to the zero value after 20 seconds, where the optimal design is obtained for the uniform electric field on Ω_p.

Detailed derivation procedure of the shape sensitivity formula and more numerical examples will be presented in the full paper.

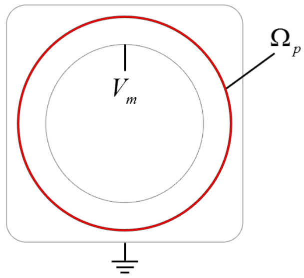

Fig. 2. Numerical test model for shape optimization of conductor surface.

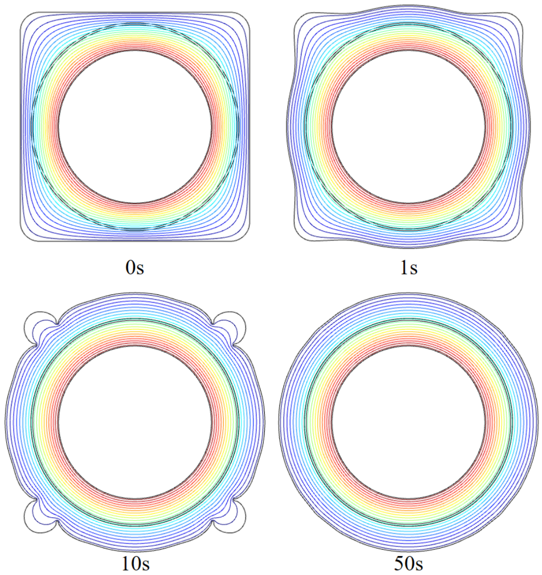

Fig. 3. Evolution of cathode shape during design process.

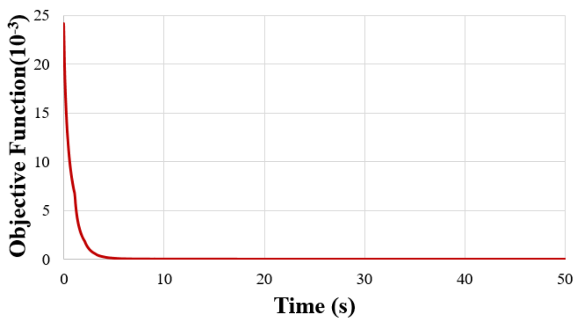

Fig. 4. Evolution of objective function during design process.

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